Lecture 5 Worksheet: Committee Problems

L5

Name:

Problem 1 (Committee Forming) *The student council has 6 first-year students, 4 second-year students, 3 third-year student, and 8 fourth-year students.*

(a) *How many ways are there to form a committee consisting of one student from each year?*

Solution: *Our decision process is to select one student from the first-years (6 possibilities), then one from the second-years (4 possibilities), then one from the third-years (3 possibilities), then one from the fourth-years (8 possibilities). Thus, our decision process reveals* $\boxed{6 \cdot 4 \cdot 3 \cdot 8}$ *total possibilities.*

(b) *How many ways are there to form a committee consisting of two students from each year?*

Solution: *Similar to last problem, our decision process is to select two students from the first-years, then two from the second-years, then two from the third-years, then two from the fourth-years. In each case, the order in which the students are selected in each committee does not matter. Thus, our decision process reveals*

$$\boxed{\binom{6}{2} \cdot \binom{4}{2} \cdot \binom{3}{2} \cdot \binom{8}{2}}$$

total possibilities.

(c) *How many ways are there to form a committee with the same number of students from each year, regardless of how many there are?*

Solution: *We must first decide how many students we'll pick from each year, then determine how many ways there are to form committees of that size. Because there are three third-year students, there can not be more than three students per year in the committee. The cases for* 1, 2, *and* 3 *students per year are entirely non-overlapping, so we can simply follow the logic from the prior two problems and add the three-student case:*

$$\boxed{6 \cdot 4 \cdot 3 \cdot 8 + \binom{6}{2} \cdot \binom{4}{2} \cdot \binom{3}{2} \cdot \binom{8}{2} + \binom{6}{3} \cdot \binom{4}{3} \cdot \binom{3}{3} \cdot \binom{8}{3}}$$

(d) *How many ways are there to form a committee consisting of two underclass (1st/2nd year) and two upperclass (3rd/4th year) students?*

Solution: *Our decision process simply first selects two underclass students and then selects two upperclass students. There are 10 underclass students and 11 upperclass students, for a total of*

$$\boxed{\binom{10}{2} \cdot \binom{11}{2}}$$

possibilities.

(e) *Each member of the student council shakes the hand of each other member exactly once. How many handshakes occur?*

Solution: *Each handshake occurs between two different people on the council. Thus, the number of handshakes is exactly the number of ways to select two different people on the council, where the order in which they are selected doesn't matter. This gives us*

$$\boxed{\binom{21}{2}}$$

handshakes.

(f) *Each member of the student council is either on the subcommittee for student life, academics, or clubs. How many possible assignments of members to subcommittees are there? (Subcommittees are allowed to be empty. No member can be in more than one subcommittee.)*

Solution: *One decision process we can use is to simply go in order by member, assigning each one to a subcommittee. At each step, the member has three possibilities for subcommittees, and there are* 21 *different members in total, which means there are* $\boxed{3^{21}}$ *possibilities.*

Problem 2 (Bonus Problem) ***Error Correction Codes*** *are a tool used to ensure that messages get transmitted properly even when the message is susceptible to errors in transmission. My messaging server does something simpler: it doesn't correct errors, but it can detect if there's been a single error in the message sent. To do so, the server has a set of "legal messages". To ensure that any single error is caught, for any length n, there are no two messages in my set that are both length n and differ by exactly one letter. For example, if "abc" is a legal message, then "abd" can not be. (This way, if I receive "abd," I know it was an error.) What is the largest number of legal n-letter messages possible? (Messages consist only of lowercase letters.)*

Solution: The largest number of legal messages turns out to be 26^{n-1} for any n-length string. One way to construct this set is as follows:

- For any string of length $n-1$:
  - Convert each letter to a number: A=0, B=1, C=2, ..., Z=25.
  - Sum up the letters in the string to get S. Take $S \mod 26$, and convert it back into a letter.
  - Use that letter as the last letter of the string.

Clearly we can do this for each string of length $n-1$, giving us 26^{n-1} different strings. We leave it as an exercise to show that (1) none of these strings are only one letter apart and (2) that you can't do any better than 26^{n-1}.