# Lecture 6 Worksheet: Counting Tricks

# L6

Name:

**Problem 1 (Tricks are for kids!)** *Each of the following problems requires at least one of the tricks from class: PIE, Complementary Counting, and Stars and Bars. Some use multiple!*

(a) *How many positive integers less than* $100$ *are a multiple of* $3$ *or* $5$*?*

(b) *How many positive integers less than* $100$ *are not a multiple of* $3$ *or* $5$*?*

(c) *A painter has been commissioned to paint 5 houses. They have three different colors to choose from, but at least one pair of adjacent houses must have the same color. How many ways are there to paint the houses?*

(d) *A painter has been commissioned to paint 5 houses. The paint shop has three different colors to choose from, and the painter buys a bucket of paint per house. How many different paint orders could be made? (Two paint orders are the same if they have the same amount of each color of paint, regardless of order)*

(e) *Ten trees need to be planted in a row. However, if the tallest tree and the shortest tree are next to one another, the shortest tree will not get any sunlight. How many ways are there to plant the trees?*

(f) *Ten trees need to be planted in a row. This row spans three blocks. How many ways are there to distribute the trees among the blocks?*

(g) *(From AIME 2002) Many states use a sequence of three letters followed by a sequence of three digits as their standard license-plate pattern. Given that each three-letter three-digit arrangement is equally likely, find the probability that such a license plate will contain at least one palindrome (a three-letter arrangement or a three-digit arrangement that reads the same left-to-right as it does right-to-left). Use complementary counting.*

(h) *Solve the previous problem using PIE.*

PROBLEM 2 (BONUS: PARKING PROBLEMS) *The first problem is harder than the course is asking for, but is doable with the tricks from this class. The second problem requires quite a bit of creativity - ask for a hint if you're stuck!*

1. *There are 20 parking spots in a row.* 14 *cars come in, one at a time, and park in a random spot. A monster truck comes in and needs two adjacent spots to park. What is the probability that the monster truck can park?*

2. *Suppose a one-way street has n parking spaces labeled* $1, 2, \cdots n$*. Each car coming in has a preferred spot, but if it's taken they will keep driving forwards and take the next open spot. If we say that the ith car wants to park in spot* $a_i$*, then call* $(a_1, a_2, \ldots, a_n)$ *a* ***parking function*** *of length n if all the cars can park. For example, the sequence* $(2, 2, 1)$ *is a parking function because car one parks in spot 2, car two can't park in spot 2 so it parks in spot 3, and car three parks in spot 1. On the other hand,* $(2, 2, 3)$ *is not a parking function because car three will try to park in spot* 3 *but it's already taken by car* 2*, and there are no more spaces after* 3*. Show that for a parking lot of size n, there are* $(n+1)^{n-1}$ *different parking functions.*