

PROBLEM 1 (GRAPH ALTERATIONS) *We've seen these algorithms work on normal graphs, but graphs often change or have unpredictable properties! In this problem, we'll explore the implications of those changes. Say you have a graph G with n vertices and m edges.*

1. *You add one edge to the graph.*

- (a) *What is the most that the distance from s to some other vertex v can increase? decrease?*
- (b) *The (undirected) graph consists of some connected components. What is the most that the number of connected components can increase? decrease?*
- (c) *Say the graph is directed. Can the number of topological orders increase? decrease?*
- (d) *Say the graph is directed, and you've split it into SCCs. What is the most that the number of SCCs can increase? decrease?*
- (e) *(Bonus) How do you find the edge that decreases the number of SCCs the most?*

2. *The graph is directed. You reverse every edge in the graph.*

- (a) *Say the original graph was a DAG. How is a topological order of the original graph related to a topological order of this new one?*
- (b) *Say the original graph had c SCCs. How many can it have now?*

3. *You take every edge in G and insert a vertex in between them, so it becomes two edges through a midpoint vertex. Each edge gets a different midpoint vertex.*

- (a) *If the original graph was bipartite, is the new graph still bipartite? What if it wasn't?*
- (b) *Say the distance from s to v was d . What is the new distance?*

4. *The graph was weighted.*

- (a) *You increase every edge weight by 1. Is the shortest path the same?*
- (b) *You multiply every edge weight by 2. Is the shortest path the same?*
- (c) *(Bonus) You multiply every edge weight by -1 . Does running Dijkstra's algorithm on this graph find you the longest path on the original graph?*

PROBLEM 2 (GRAPH MODELING PRACTICE) For each of the following problems, provide a solution that involves modeling it as a graph and running one of our known algorithms some number of times. You'll need to state the following:

- What are the vertices in your graph?
 - What are the edges in your graph?
 - Which algorithm would you use here? How would you use it?
 - How do you use the result of the algorithm to return the answer?
1. You want to split the class into two teams to run a debate, but you know some students are in messy situationships and want to make sure they're split up. Determine whether it's possible to split the students up.
 2. (Bonus) What if you also know some students are best friends and want to be together?
 3. You've made a new social networking app called "Convrs". Users on Convrs can have asymmetric following. A group chat can only be formed if, for every two people a, b in the chat, there is a "follow chain" from a to b (a follows b , or a follows someone who follows b , etc). For any group chat, determine quickly whether this is possible.
 4. You want to go from school to your friend's house, but you need to stop by the library to pick up a book first. Find the shortest path from school to your friend's house that passes through the library.
 5. (Bonus) Try it with only one run of Dijkstra's!
 6. A person's "Erdős number" is the distance they are from collaborating with prolific mathematician Paul Erdős. For example, if a scholar has co-authored a paper with Paul Erdős in a movie, they have a Erdős number of 1. If they've co-authored a paper with someone who co-authored with Erdős, they have a Erdős number of 2. Given a database of all papers and authors, find everyone's Erdős number.
 7. (Very Bonus) You want to go on a roadtrip from A to B , but you're driving a new futuristic electric car that replaces its battery at every battery station. Furthermore, different batteries can go different distances! You're starting at battery station A , and you're given a map that contains all the different battery stations, roads that connect them (with distances), and the distance that the battery at each station can take you. Find the shortest achievable path from A to B , and the batteries you'd need to take. Remember you can get to a station but skip its battery if your current battery can take you to the next station as well.