L22

Problem 1 (Practice with Reductions) *For each of these pairs of problems, describe a reduction from one problem to the other. Then, write "[Problem A] reduces to [Problem B]. Therefore, [Problem A] is no harder than [Problem B]."*

- From the homework. Be super careful with your conclusion here!
  - **Minimum Spanning Tree:** Given as input a graph G with weighted edges, find a minimum spanning tree.
  - **Teleporters:** Given as input a graph G with weighted edges, where some vertices function as teleporters, connect the whole graph.

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  - **Gift Packing:** Given as input a set of presents with weight $w_1, w_2, \ldots, w_n$ and a fixed size V, output a subset that weighs exactly V in total. (B)
  - **Truck Balancing:** Given a set of boxes $w_1, w_2, \ldots, w_n$, find a way to split them into two subsets that weigh exactly the same amount. (A)

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  - **2-Coloring:** Given as input a graph $G = (V, E)$, determine whether it is possible to assign all vertices to either red or blue, so that no edge connects two vertices of the same color.
  - **3-Coloring:** Given as input a graph $G = (V, E)$, determine whether it is possible to assign all vertices to either red, blue, or green, so that no edge connects two vertices of the same color.

PROBLEM 2 (CERTIFICATES) *For each of the following problems, prove that it is in NP by specifying the certificate and describing how it can be used to verify the claim.*

(a) ***Truck Balancing:*** *Given a set of boxes $w_1, w_2, \ldots, w_n$, is there a way to split them into two subsets that weigh exactly the same amount?*

(b) ***Course Scheduling:*** *Given a set of courses, can you take 6 courses this semester?*

(c) ***3-Coloring:*** *Given as input a graph $G = (V, E)$, is it possible to assign all vertices to either red, blue, or green, so that no edge connects two vertices of the same color?*

(d) ***Compositeness:*** *Given an integer, is it composite?*