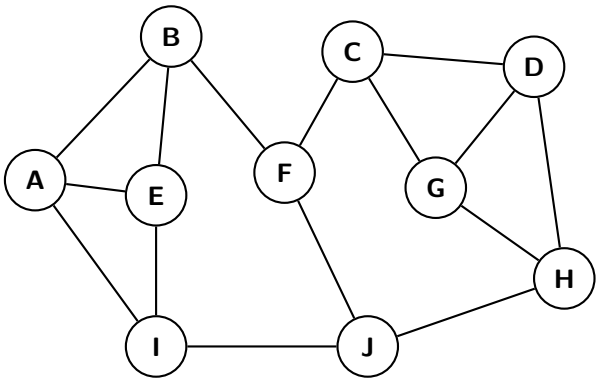
Problem 1 (Practice with Reductions) *Find the largest independent set of this graph. Then, find the smallest vertex cover of the graph. Finally, design a reduction from one problem to the other.*



Problem 2 (3SAT Practice) *Consider the following formula on four variables x_1, x_2, x_3, x_4:*

$$\phi = (x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_2 \vee x_4) \wedge (x_1 \vee x_3 \vee \overline{x_4}).$$

(a) Does the following assignment satisfy the formula? $x_1 = T, x_2 = T, x_3 = T, x_4 = F$.

(b) Find an assignment that satisfies the formula.

(c) Find another.

Problem 3 (Using 3SAT to Represent Other Logical Statements) *Of course, 3SAT is only an interesting problem if it can be used to represent other things we care about. This type of formula feels like it might be constraining though. Let's see what we can do with it.*

(a) Given variables x, y, z, write a formula that can only be satisfied if **at least one** variable is true.

(b) Given variables x, y, z, write a formula that can only be satisfied if **at most one** variable is true.

(c) The following formula can only be true when $x = y$.

$$\phi = (x \vee \overline{y} \vee z) \wedge (x \vee \overline{y} \vee \overline{z}) \wedge (\overline{x} \vee y \vee z) \wedge (\overline{x} \vee y \vee \overline{z}).$$

Why? [Hint: If z is true, which clauses need to be satisfied? What if z is false?]

(d) Write a formula that can only be true when $x \neq y$.