

Pseudocode

Here is a version of pseudocode for the algorithm, for reference.

Algorithm 1 Gale-Shapley (1962)

```

1: procedure STABLEMATCHING(Doctor Preferences, Hospital Preferences)
2:   final_pairs  $\leftarrow$  empty array
3:   while there is an unmatched hospital  $h$  do
4:      $h$  makes an offer to their favorite doctor  $d$  (that they haven't requested before)
5:     if  $d$  has no prior offer then
6:       Add  $(d, h)$  to final_pairs
7:     else
8:       If  $d$  prefers  $h$  to its current pairing  $h'$ , replace  $(d, h')$  in final_pairs with  $(d, h)$ .
9:     end if
10:  end while
11:  return final_pairs
12: end procedure

```

PROBLEM 1 (STABLE MATCHING WARMUP) Here is the outline for a proof that the algorithm returns a stable matching. Fill in the blanks.

Proof. Assume, for contradiction, that the returned matching is not stable. Then there exists some pair (d, h) that is blocking. For this pair to not be in the final matching, it either was never requested by h , or

If it was never requested, then by line __, h has only made requests to doctors it prefers over d , so h must prefer the doctor it is matched with, so this is not a blocking pair.

If (d, h) was requested but is not in the final matching, then by line __, _____, so it is not possible for this to be a blocking pair.

This contradicts the claim that there is a blocking pair (d, h) .

□

PROBLEM 2 (BIG-O PRACTICE) Fill in the table by first finding the Big-O bound for each of these runtimes, then by sorting from fastest (Rank=1) to slowest (Rank=5).

Runtime	Big-O	Rank
$9n^3 + 7n^2 + n^8$		
$14n + 29$		
$n \log n + 3n$		
$3^n + 7n^9$		
123456789		

PROBLEM 3 (INTUITION CHECK) Why are the proposers happier?