

PROBLEM 1 (ADDITION) *Can we do better? Why or why not?*

PROBLEM 2 (MULTIPLICATION) *For each of the following algorithms, as well as the one on the board, take a guess about which one(s) are fastest. Then, analyze the runtime, given that addition takes the amount of time described in the previous problem.*

Algorithm 1 Multiplication on Binary Numbers of Length n

```

1: procedure MULTIPLY( $x, y$ )
2:   if  $y = 0$  then
3:     return 0
4:   end if
5:    $z \leftarrow \text{MULTIPLY}(x, \lfloor y/2 \rfloor)$ 
6:   if  $y$  is even then
7:     return  $2z$ 
8:   else
9:     return  $x + 2z$ 
10:  end if
11: end procedure

```

Algorithm 2 to multiply x and y , with an example given for $x = 11, y = 13$:

Write x and y next to each other	11	13
Divide the first number by 2 (rounding down), double the second number	5	26
Repeat until the first number is 1.	2	52
	1	104
Add the numbers on the right where the left number is odd.	$13 + 26 + 104$	$= \boxed{143}$

Runtime for Algorithm 0 (on the board): _____ Algorithm 1: _____ Algorithm 2: _____

PROBLEM 3 (FASTER MULTIPLICATION) *Follow the procedure on the board to multiply 78×13 . How many multiplications did you do? How many would you have done following any of the other algorithms?*