MATH 1B DISCUSSION WORKSHEET - 11/6/18

POPULATIONS AND LINEAR DIFFERENTIAL EQUATIONS

1. Linear Differential Equations

(1) Determine whether the following differential equations are separable, linear, or both. Then, solve each one.

(a) $xy' + 2y = x^2 - x$

(b) $y' + 3xy = 6x$

(c) $y' + 3xy = 6$

(d) $xy' - 2y = -8$

(e) $y' - \frac{1}{2}y = \sin(2t)$

(f) $xy' + 2y = 12x^2$

(2) Using the last differential equation, find a solution for

$$xy'' + 2y' = 12x^2.$$

(3) Show that for some **homogeneous** differential equation

$$\frac{dy}{dx} + p(x)y = 0,$$

if $y = f(x)$ and $y = g(x)$ are both valid solutions, then $y = f(x)+g(x)$ is a valid solution as well.

2. Population Models

(1) Suppose we have a pond that will support 1000 fish, and the initial population is 100 fish. Write out a logistic differential equation for this scenario, and determine its solution.

(2) Psychologists in learning theory study learning curves, the graphs of the performance function $P = P(t)$ of someone learning a skill as a function of the training time t. If M represents the maximal level of performance, it is noted that for a certain skill the learning is at first rapid, and then it tapers off (the rate of learning decreases) as $P(t)$ approaches M.

(a) Explain why solutions to the differential equation $\frac{dP}{dt} = k(M - P)$, k a positive constant, fulfill the above description of P. Sketch a typical learning curve.

(b) What's the relationship between this differential equation and the logistic population growth model?

(c) (c) Suppose that, for a specific learning activity, it is determined that $P_0 = .1M$ and $k = 0.05$ for t measured in hours. How long does it take to reach 90% of M, the maximal level of performance?