MATH 1B MOCK FINAL

BUT SHORTER AND WITHOUT CHAPTER 17

(1) A 400L swimming pool starts currently holds 200 liters of water as well as 500 milligrams of chlorine. Clean water flows in at a rate of 5 liters per minute, and the mixed water flows out at 3 liters per minute.

Swimming pools are considered safe when the chlorine concentration is below 1mg/L. Will the swimming pool reach a safe level of chlorine before it overflows? Justify your answer.

(2) Evaluate the integral

$$\int_1^\infty \frac{dx}{x^2(x+2)},$$

or demonstrate that the integral diverges.

(3) Determine whether each of the following series converges or diverges.

(a) $0.9 - 0.99 + 0.999 - 0.9999 + 0.99999 \cdots$

(b) $\dfrac{1}{2^2} + \dfrac{2}{3^2} + \dfrac{3}{4^2} + \cdots$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2 + 2n}{n^3 + 1} - \frac{1}{2} \right)^n$

(4) Solve the following differential equations.
   (a) $xy' - y = x \ln x$

   (b) $y' = 0.1y - 0.00005y^2$

(5) Find the Taylor series of the following functions:

$$(a)\ f(x) = \frac{1}{\sqrt{x}}$$

$$(b)\ f(x) = 1 - \sin^2(x)$$

(6) Find the curve that passes through the point $(3, 2)$ and has the property that if the tangent line is drawn at any point P on the curve, then the part of the tangent line that lies in the first quadrant is bisected by P. In the diagram below, you can see what it should look like: the two parts of the line separated by the point P are of equal length in the first quadrant. (Hint: first find the slope of the line through the point $(3, 2)$ whose part in the first quadrant is bisected by $(3, 2)$. How can we generalize this process?)