

MATH 1B DISCUSSION WORKSHEET - 11/15/18

COMPLEX NUMBERS

1. COMPLEX NUMBER ARITHMETIC

Rewrite the following as complex numbers of the form $a + bi$, where a and b are both real numbers.

(1) $\overline{-3 + 2i}$

(2) $(2 + 3i) + (3 + 5i)$

(3) $(2 + 3i)(3 + 5i)$

(4) $\frac{i-4}{2i-3}$

(5) $e^{\ln 2 + i\frac{\pi}{4}}$

Rewrite the following as complex numbers of the form $re^{i\theta}$, where r is a real number and θ is a number between 0 and 2π , inclusive.

(1) $2 - 2i$

(2) $-3 + 3\sqrt{3}i$

(3) $(1 + i)^6$

(4) $(-2 + 2i)(-3 - \sqrt{3}i)$

(5) $\sqrt{-1 + i}$

2. IMPORTANT TOOLS WITH COMPLEX NUMBERS

2.1. **Polynomial Roots.** Find the roots of the following polynomials in either Polar or Cartesian form.

(1) $x^2 + 1$

(2) $x^2 + 2x + 3$

(3) $2x^2 + 6x + 7$

(4) $x^n + 1$ for any natural number n . These are called the n th roots of unity.

(5) $x^8 - 1$

2.2. **Proofs.**

(1) Prove that $\overline{wz} = \overline{w} \cdot \overline{z}$ for any complex numbers w and z .

(2) Prove that there exists no complex number z such that $|z| - z = i$.

(3) Find, with proof, the number of solutions to the equation $(a + bi)^{2002} = a - bi$.