

## MATH 1B DISCUSSION WORKSHEET - 9/6/18

### RIEMANN SUMS AND ERRORS

#### 1. THINGS TO KNOW FOR YOUR QUIZ! (IN TERMS OF APPROXIMATIONS ANYWAY)

#### 2. FINDING ERROR BOUNDS (IMPORTANT)

Find the error bounds for the following functions and intervals for Midpoint, Trapezoidal, and Simpson's.

- (1)  $\int_{-2}^5 (-2x^3 + 7x^2 + 5x - 12) dx$ . Partitions of length 1. [What's the error for Simpson's? What does that tell us about Cubics?]
- (2)  $\int_0^4 \left(\frac{-1}{24}(x-2)^4 + x^3\right) dx$ . Partitions of length 1.
- (3)  $\int_{-0.5}^{0.5} \sin(\theta^2) d\theta$ . Partitions of length 0.1. (Hint: these angles aren't that big...)

#### 3. BUILDING ERROR INTUITION

For each of the following functions, rank the five types of approximations in terms from lowest to highest, or state that it can't be properly ranked without knowing more about the partitions. Assume all partitions are regular partitions unless stated otherwise.

The five types of approximations are Left Endpoint, Right Endpoint, Midpoint, Trapezoidal, and Simpson's.

- (1)  $f(x) = x^2$  for  $x$  in  $[0, 10000]$ .
- (2)  $f(x) = \sin(x)$  for  $x$  from 0 to  $\pi/2$ .
- (3)  $f(x) = \sin(x)$  for  $x$  from  $\pi/2$  to  $\pi$ .
- (4)  $f(x) = \sin(x)$  for  $x$  from 0 to  $2\pi$ .

## 4. UNFORTUNATE RIEMANN SUMS

- (1) Is it possible to find a partition where both left endpoint approximation and right endpoint approximation give underestimates, but Midpoint Rule gives an overestimate?
- (2) Is it possible to find a partition where both left endpoint approximation and right endpoint approximation give underestimates, but Trapezoidal Rule gives an overestimate?
- (3) Draw a partition in which left endpoint approximation, right endpoint approximation, and Trapezoidal Rule would provide significantly less error than Midpoint Rule.
- (4) Draw a partition in which left endpoint approximation and Midpoint Rule would both provide very little error, but Trapezoidal Rule would give a significant amount of error.

## 5. ERROR BOUND EXPLORATION (PRETTY CHALLENGING)

We never actually went over what the error bounds would look like for left endpoint and right endpoint approximations. It probably isn't surprising to you that they're the same. Can you figure out what they are?

(Hint: the form is pretty similar to our previous error bounds, Furthermore, we're choosing  $K$  such that  $|f(x)| \leq K$  (that's right, no derivative).)