# MATH 1B DISCUSSION WORKSHEET - 9/13/18

ARC LENGTH, SURFACE AREA, AND MASSES

## 1. Comparisons and p-values

Use the Comparison Theorem to determine whether each integral is convergent or divergent.

(1) $\int_1^\infty \sqrt{1+\frac{1}{x^3}}dx$
(2) $\int_0^\infty \frac{dx}{x+e^x}$
(3) $\int_0^\infty \frac{3x+2\sin(x)}{x^3+5}\,dx$
(4) $\int_0^\infty 2^{\left(-x^2\right)}\,dx$

## 2. Arc Length

[Exercise from Pauls Online Notes]

(1) Find the length of $x = \left(\frac{3y}{2}\right)^{2/3} + 1$ between $0 \leq y \leq 2\sqrt{3}$.

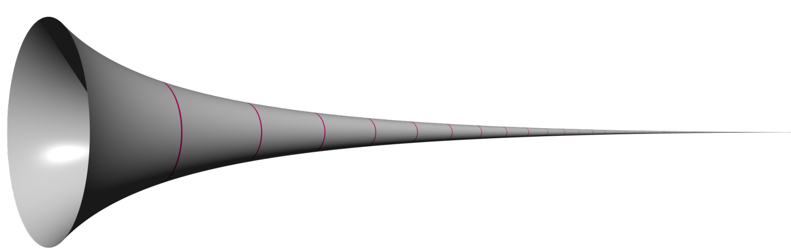
(2) Redo the previous problem in the form $y = f(x)$ instead. (Rewrite the previous equation to in terms of $y = f(x)$, look at what range the value of $x$ changes by in that span of y-values, and find the arc length again.) Do we expect the answers to be the same?

## 3. Surface Area

(1) Find the surface area of the rotation of the curve $y = 4-x$ for $1 \leq x \leq 3$ about the $x$-axis.

(2) Find the surface area of the rotation of the curve $y = 4-x$ for $1 \leq y \leq 3$ about the line $y = -2$.

(3) Now that you know how to find surface area, we can look at Gabriel's Horn one more time. Gabriel's Horn is defined as the rotation of the curve $y = \frac{1}{x}$ for $1 \leq y \leq \infty$ about the $x$-axis.



(a) Find the length of the curve. (This one is pretty obvious.)
(b) Find the volume.
(c) Find the surface area.

## 4. Centers of Mass

Finding the Center of Mass of a region bounded by $f(x)$ on top and $g(x)$ below consists of these steps:

- First, find the moments $M_x$ and $M_y$. These denote the tendency of the region to rotate around the $x$ or $y$ axis, respectively. Their equations are:
$$M_x = \frac{1}{2}\int_a^b y \cdot f(x) - y \cdot g(x)\, dx = \frac{1}{2}\int_a^b (f(x))^2 - (g(x))^2\, dx$$
$$M_y = \int_a^b xf(x) - xg(x)\, dx$$
- Solve for the total mass $M$. This is simply the area between the two curves.
- The center of mass, $(\overline{x}, \overline{y})$ can be found with:
$$(\overline{x}, \overline{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right).$$

(1) Find the center of mass of the semicircle made by $y = \sqrt{4 - x^2}$ and the $x$-axis.

(2) Determine the center of mass of the region bounded by the parabola $y = x^2$ and the line $y = 9$.