MATH 1B DISCUSSION WORKSHEET - 9/18/18

CHAPTER 8 REVIEW SHEET

1. Arc Length

1.1. **What is it?** The length of the curve.

1.2. **Formulas to know.**

- $L = \int_a^b ds$
- $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$
- $ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$
- $s(t) = \int_1^t \sqrt{1 + [f'(x)]^2}\, dx = \int_1^t \sqrt{1 + [g'(y)]^2}\, dy$

1.3. **Things to Remember.**

- We'll almost certainly give you integrals that aren't impossible to evaluate. It's pretty likely that the term inside will be a perfect square; if not, use trig sub.
- We can substitute in both x and y for any arc. If you switch orders, make sure your bounds are still proper! $1 \leq x \leq 5$ is often a very different bound from $1 \leq y \leq 5$.

2. Area of the Surface of a Revolution

2.1. **What is it?** The Surface Area of the shape formed when we perform the revolution of a curve around a line.

2.2. **Formulas to know.**

- Rotation about the line $y = c$ (Parallel to the x-axis): $S = \int_a^b 2\pi|y - c|\, ds$
- Rotation about the line $x = d$ (Parallel to which axis?): $S = \int_a^b 2\pi|x - d|\, ds$

2.3. **Things to Remember.**

- Make sure to check that the curve never crosses the axis that we're rotating across, or else it must be split up.
- The fact that ds can change actually means that for each surface we have two separate means of integration!

3. Hydrostatic Force and Pressure

3.1. **What is it?** We're finding a way to measure the entire force of water on a submerged object.

3.2. **Formulas to know.**

- $F = \int PA$ [F = Force, P = Pressure, A = Area]
- $P = \rho g d$ [P = Pressure, ρ = Density of fluid, g = force of gravity, d = depth (distance between the top of the water and the layer)]
- $d =$ [Water Height] $- y$
- $A = 2$(Width at height y) dy

4. Moments and Centers of Mass

4.1. **What is it?** The Moment of an object is its tendency to rotate around some point or axis. The center of mass is self-explanatory.

4.2. **Formulas to know.**

- $A = \int_a^b f(x) - g(x)$
- $M_x = \int_a^b xf(x) - xg(x)\,dx$
- $M_x = \frac{1}{2}\int_a^b [f(x)]^2 - [g(x)]^2\,dx$
- $(\overline{x}, \overline{y}) = \left(\frac{M_y}{A}, \frac{M_x}{A}\right)$
- Theorem of Pappus: If there exists some region R rotated around some line ℓ where R does not cross ℓ, then the total volume of the resultant revolution is equal to the distance traveled by the centroid multiplied by the area of region R.

5. Consumer Surplus

5.1. **What is it?** Most people like a product enough that they would pay more for it than what is charged for it. The sum of all that money that those people are "saving" by not paying as much as they'd be willing to is called the **Consumer Surplus**.

5.2. **Formulas to know.**

- P = p(x) [P = Price that the product is actually sold at, p = the function that outputs the price for a given quantity x, x = quantity bought/sold.]
- Consumer Surplus $= \int_0^X p(x) - P\,dx$, where X = quantity sold in total

6. Blood Flow

6.1. **What is it?** Amount of flow of blood through a vein.

6.2. **Formulas to know.**

- Flux (F) = $\frac{\pi P R^4}{8\eta l}$

7. Cardiac Output

7.1. **What is it?** The rate at which the heart outputs blood.

7.2. **Formulas to know.**

- Flow (F) = $\frac{A}{\int_0^T c(t)dt}$

7.3. **Things to Remember.** Because this flow is found through sampling data, it's typical that this problem requires you to approximate the integral in the denominator.

8. Probability Density Functions

8.1. **What is it?** A probability density function is a function that helps tell us the probability that a random variable sampled falls within a certain range.

8.2. **Formulas to know.** For some probability density function f on a distribution X,

- $P(a \leq X \leq b) = \int_a^b f(x)\,dx$
- $\int_{-\infty}^{\infty} f(x)\,dx = 1$
- $f(x) \geq 0$ for all x.

8.3. **Things to Remember.** Make sure to check the bottom two conditions to determine whether or not a function is a PDF.

9. Average Values

9.1. **What is it?** We want to find a way to use the PDF $f(x)$ to determine the expected (average) value we get when we sample from the distribution.

9.2. **Formulas to know.** For some probability density function f on a distribution X,

- $\mu = \int_{-\infty}^{\infty} xf(x)\,dx$

10. Normal Distributions

10.1. **What is it?** A special, extremely common distribution that comes up very often.

10.2. **Formulas to know.** A normal distribution with mean μ and standard deviation σ has a PDF of

- $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$