# MATH 1B DISCUSSION WORKSHEET - 9/27/18

TALKING ABOUT SEQUENCES!

## 1. Important Terminology

**Definition 1.1** (Boundedness)**.** A sequence $\{a_n\}$ is **bounded above** if there is a number $M$ such that $a_n < M$ for all $n \geq 1$. A sequence $\{a_n\}$ is **bounded below** if there is a number $M$ such that $a_n > M$ for all $n \geq 1$. A sequence $\{a_n\}$ is **bounded** if it is *both* bounded above and bounded below.

(1) What can we say about the graph of any bounded sequence?

(2) Is the sequence modeled by $a_n = \sin(n)$ bounded above? Bounded below? Bounded?

**Definition 1.2** (Monotonicity)**.** A sequence $\{a_n\}$ is **increasing** if for every $n$, $a_{n+1} > a_n$. A sequence $\{a_n\}$ is **decreasing** if for every $n$, $a_{n+1} < a_n$. A sequence is **monotonic** if it is either increasing or decreasing.

(1) Let's say we have a sequence $\{a_n\}$ where $a_n = f(n)$ for some function $f$. What does the behavior of the derivative of $f$ tell us about the monotonicity of $\{a_n\}$? (For example, if $f'(x) < 0$ for all $x$, what can we say about $\{a_n\}$?)

**Definition 1.3** (Limit of a sequence)**.** A sequence $\{a_n\}$ has the limit $L$ and we can say that $\lim_{n\to\infty} a_n = L$ if for every $\epsilon > 0$ there is a corresponding integer $N$ such that if $n > N$, then $|a_n - L| < \epsilon$.

(1) Intuitively or graphically, what does $|a_n - L|$ represent?

(2) In that case, what does $|a_n - L| < \epsilon$ mean?

(3) What, then, does "if $n > N$, then $|a_n - L| < \epsilon$" mean?

(4) Rewrite the entire definition of a limit in a way that's easier for you to understand.

**Definition 1.4** (Infinite Limit)**.** If we say that $\lim_{n\to\infty} a_n = \infty$, this means that for every positive number $M$ there is a corresponding integer $N$ such that if $n > N$, then $a_n > M$.

(1) Building off our work from the previous problem, rewrite this definition to make it more intuitive for you as well.

## 2. Limit Rules

If $\lim_{n\to\infty} a_n = L_a$ and $\lim_{n\to\infty} b_n = L_b$, and $k$ is some constant, then

(1) $\lim_{n\to\infty}(a_n + b_n) =$

(2) $\lim_{n\to\infty}(a_n - b_n) =$

(3) $\lim_{n\to\infty}(a_n * b_n) =$

(4) $\lim_{n\to\infty}(a_n \div b_n) =$

(5) $\lim_{n\to\infty} k(a_n) =$

(6) $\lim_{n\to\infty}(a_n)^k =$

(7) If $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} |a_n| =$

(8) If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n =$

(1) What must be true for us to properly use equation (4)?

(2) Equations (7) and (8) aren't always true if the limit was some non-zero number. Find counterexamples for them.

## 3. A single all-inclusive example

Consider the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2a_n}$.

(1) Write out $a_2$, then find its value by plugging into a calculator.

(2) Write out $a_3$, then find its value by plugging into a calculator.

(3) Show that $a_n > 1$ for all $n$.

(4) Show that $a_n < 2$ for all $n$.

(5) The previous two proofs show that $\{a_n\}$ is ...

(6) Use the results from (3) and (4) to demonstrate that $a_{n+1} > a_n$ for all $n$.

(7) This demonstrates that $\{a_n\}$ is ...

(8) If statements (5) and (7) are true, then $\{a_n\}$ is ...

(9) (Bonus) Show that $\lim_{n\to\infty} a_n = \sqrt{2\sqrt{2\sqrt{2\sqrt{2...}}}} = 2$.