MATH 1B DISCUSSION WORKSHEET - 10/4/18

COMPARISON TEST QUESTIONS

Determine whether the following series converge or diverge. You may use Comparison Test, Limit Comparison Test, Divergence Test, p-Test, or the fact that geometric series always converge when the ratio is less than 1.

(1) $\sum_{n=1}^{\infty} \frac{3n}{n^2-8}$

(2) $\sum_{n=1}^{\infty} \frac{n+1}{n^3+n}$

(3) $\sum_{n=1}^{\infty} \frac{(n)(n+1)}{(n+2)(n+3)}$

(4) $\sum_{n=1}^{\infty} \frac{e^{-n}}{2n^2+3n+5}$

(5) $\sum_{n=1}^{\infty} \frac{3n^3-7}{\sqrt{n^7+6n^2}}$

(6) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-n}$

(7) $\sum_{n=1}^{\infty} \frac{1}{n!}$

Bonus: We know that the Harmonic Series $H_n = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges. The Kempner Series is the series composed of the sum of the terms of the Harmonic Series that do not contain a 9. For example, the terms $\frac{1}{9}, \frac{1}{19}, \frac{1}{29}, \frac{1}{99}$, and $\frac{1}{397}$ and so on are all omitted from the sum. Prove that the Kempner Series converges using the following steps.

(1) Prove that the number of terms in the Kempner Series with n digits in the denominator equal to $8(9^{n-1})$.

(2) Prove that any term with n digits in the denominator is less than $\frac{1}{10^{n-1}}$.

(3) Demonstrate that the Kempner Series follows the following inequality:

$$\sum_{n=1}^{\infty} K_n \leq \sum_{n=1}^{\infty} 8(9^{n-1})\frac{1}{10^{n-1}}$$

(4) Use this inequality and the Comparison Test to determine that the Kempner Series converges, and find an upper bound for its sum.