MATH 1B DISCUSSION WORKSHEET - 10/18/18

TAYLOR AND MACLAURIN SERIES

Fill out the following chart.

Function	Maclaurin Series	First Few Terms	Radius
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	$1 + x + x^2 + x^3 + \cdots$	$R = 1$
e^x			
$\sin(x)$			
$\cos(x)$			
$\tan^{-1}(x)$			
$\ln(1+x)$			
$(1+x)^k$			

(1) Find the Taylor series expansion of $\ln(x)$ centered at $x = 4$.

(2) Find the Taylor series expansion of $\sqrt{x}$ centered at $x = 9$.

(3) Find the Maclaurin series expansions of the following series by using the table above:
(a) xe^x

(b) $\frac{d}{dx}\left[x^3 \cos(2x)\right]$

(c) $\int \frac{e^{3x} - e^{2x}}{x}\ \mathrm{dx}$

(4) Find the Maclaurin series expansion of $\ln(1+x^2)$, and determine its radius of convergence.

(5) Using Maclaurin series expansions, find

$$\lim_{n\to 0} \frac{\sin(x) - x + \frac{1}{6}x^3}{x^5}$$

(6) Using the Maclaurin series expansions, determine the values of

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(\ln x)^n}{n!}$

(b) $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots$

(7) (Bonus) If we have to prove that Taylor series expansions actually work, then there must be some functions which don't allow for a proper approximation! For each of the following functions, explain why a Taylor series expansion wouldn't work.

(a) $f(x) = \begin{cases} x^2 & x \geq 0 \\ x - 1 & x < 0 \end{cases}$ centered at x=0.

(b) $f(x) = |x|$ centered at $x = 0$.

(c) $f(x) = \begin{cases} (x-1)^2 + 1 & x \geq 1 \\ (x-1)^3 + 1 & x < 1 \end{cases}$ centered at x=1.

(d) $f(x) = e^{-\frac{1}{x^2}}$