MATH 1B DISCUSSION WORKSHEET - 10/16/18

POWER SERIES AND FUNCTION REPRESENTATION

(1) Find the interval and radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(x-6)^n}{n^n}.$$

(2) Find the interval and radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{2^n}{n}(3x-6)^n.$$

(3) Find the interval and radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{1}{(-3)^{2+n}\,(n^2+1)}(4x-12)^n.$$

(4) Write the given functions by their power series and find the interval of convergence.
   (a) $\frac{3x^2}{4-2x}$

   (b) $\frac{1}{(1+x)^2}$ [What's the integral of this fraction?]

   (c) $\ln(10-2x)$

(5) Find the sum of the alternating harmonic series!
   (a) Find the derivative of $\ln(1+x)$, and find its derivative's power series.

   (b) Use this to find the power series of $\ln(1+x)$.

   (c) What's the sum of the alternating harmonic series?

(6) Note that e^x is a positive, nonzero function whose derivative is itself. Furthermore, the nth derivative of e^x is itself as well for any positive integer n. Find the power series representation of e^x by finding the only power series satisfying the properties listed above.

(7) (Bonus) Find the power series representations for $\sin(x)$ and $\cos(x)$ by noting the following properties:
   - $(\sin(x))'' = -\sin(x), (\cos(x))'' = -\cos(x)$.
   - $\sin(x)$ is an odd function, while $\cos(x)$ is an even function.
   - $(\sin(x))' = \cos(x)$, $(\cos(x))' = -\sin(x)$.

(8) (Extra bonus for overachievers) Find the value of

$$\frac{1}{\sqrt{2}} + \frac{4}{2} + \frac{9}{2\sqrt{2}} + \frac{16}{4} + \cdots$$