# MATH 1B MIDTERM 2 MOCK EXAM

(1) Use the Test for Divergence to demonstrate that

$$\lim_{n\to\infty} \frac{(2n)^{2n}}{(3n)!} = 0.$$

(2) Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n^2 + e^n}{1 + (2n)^n}$$

(3) Determine whether the following series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(\sqrt{n})}$$

(4) Find the first three nonzero terms of the Taylor Series approximation for

$$f(x) = \sin(x)e^{-2x}$$

centered around $x = 0$, and use Taylor's Inequality to determine this approximation's maximum error on the interval $[-1, 1]$.

(5) Determine the interval of convergence of the Taylor Series

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x-1)^{4n+1}}{(2n)!}.$$

Then, determine the function modeled by this Taylor Series within the interval of convergence. [Hint: Integrating the series might help!]

(6) (Extra Credit) Prove that if $a_n \geq 0$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ converges then $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$ converges.